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PHYSICS BASED ALGORITHMS FOR FUZZY TRANSPORTATION PROBLEM.

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ABSTRACT

Transportation problems are computationally challenging due to their nondeterministic polynomial-time hard (NP-hard) nature, especially when uncertainties in costs, supply, and demand are involved. The fuzzy transportation problem addresses these uncertainties by representing them as fuzzy numbers, requiring robust methods for ranking and solving optimization models. Traditional exact methods become impractical for such problems, leading to the adoption of metaheuristic approaches. This study introduces three novel physics-based algorithms: one single-point-based and two population-based methods. These algorithms leverage principles from gravitational attraction, electromagnetism, and water flow dynamics, incorporating innovative neighborhood structures tailored to the fuzzy nature of transportation problems. The proposed methods were tested on diverse problem sizes and compared against established optimization techniques, such as genetic algorithms and commercial software, using Python implementations. An experimental design methodology was employed to optimize parameter tuning, enhancing computational efficiency and reducing experimentation time. Results demonstrate that the physics-based algorithms achieve competitive performance in terms of solution quality, computational efficiency, and adaptability to uncertainty. These findings underscore the potential of physics-based methods to advance optimization in fuzzy transportation and other domains with inherent uncertainties, paving the way for further research in hybrid and multi-objective approaches.

Keywords: *Fuzzy Transportation Problem; Physics-Based Algorithms; Metaheuristic Optimization; Gravitational Search Algorithm; Electromagnetism-Like Algorithm; Intelligent Water Drops Algorithm.*

INTRODUCTION

In today's competitive marketplace, organizations are under pressure to deliver products to customers in a timely and cost-effective manner. One of the critical challenges they face is optimizing transportation networks to minimize costs while satisfying customer demand. This problem is typically modeled as a Transportation Problem (TP), which seeks to determine the most cost-efficient way to move goods from suppliers to consumers, given certain constraints (Alatas & Can, 2015). However, real-world transportation systems are subject to uncertainties in costs, resource availability, and demand. These uncertainties make classical transportation models less effective in practice.

To address these uncertainties, researchers have turned to fuzzy transportation models (FTPs), which allow transportation costs, supply, and demand to be represented as fuzzy numbers. Fuzzy numbers can model imprecision, making them ideal for scenarios where exact values are not available or fluctuate (Birbil & Fang, 2003). FTPs have become essential in modeling real-world problems, especially in logistics and supply chain management. However, solving FTPs efficiently is challenging due to the NP-hard nature of these problems, requiring advanced solution techniques (Duan *et al.*, 2008).

The fuzzy transportation problem (FTP) extends the classical transportation model by introducing fuzzy parameters that capture uncertainty in costs and quantities. Traditional deterministic methods struggle to handle these complexities effectively. As a result, there has been growing interest in the use of metaheuristic algorithms**—**high-level problem-solving frameworks designed to find near-optimal solutions for complex optimization problems (Asi & Dib, 2010). These algorithms have been successfully applied to TPs and other optimization problems with uncertainty, providing robust and flexible solutions (Duman *et al.*, 2010).

Metaheuristic algorithms, such as genetic algorithms (GA)**,** simulated annealing (SA)**,** and particle swarm optimization (PSO)**,** have been widely used to solve FTPs due to their ability to explore large solution spaces and escape local optima (Han *et al.*, 2018). However, there are limitations to these approaches, including slow convergence rates and the need for careful parameter tuning (Rather & Bala, 2020). These challenges highlight the need for more innovative algorithms that can efficiently address the complexities of FTPs.

Physics-based algorithms have emerged as a promising alternative to traditional metaheuristics. These algorithms are inspired by natural physical processes, such as gravity, electromagnetism, and water flow dynamics, to solve optimization problems (Alatas & Can, 2015). Some notable physics-based algorithms include **the** Gravitational Search Algorithm (GSA)**,** the Electromagnetism-like Algorithm (EMA)**,** and the Intelligent Water Drops Algorithm (IWD)**.** These algorithms leverage the principles of physical phenomena to navigate the solution space and find optimal or near-optimal solutions.

For instance, the Gravitational Search Algorithm (GSA) models the behavior of objects under the influence of gravity, with solutions being attracted to one another based on their fitness (Rashedi *et al.*, 2009). Similarly, the Electromagnetism-like Algorithm (EMA) simulates the attraction and repulsion between charged particles, guiding solutions toward optimal regions (Birbil & Fang, 2003). The Intelligent Water Drops Algorithm (IWD), inspired by the natural flow of water in rivers, uses the concept of erosion to identify optimal paths (Shah-Hosseini, 2009).

These algorithms have proven effective in various optimization problems, including those involving continuous and discrete variables (Chatterjee *et al.*, 2010; Rabanal *et al.*, 2008). However, their application to fuzzy transportation problems remains largely unexplored. Given the success of physics-based algorithms in other domains, there is significant potential for these methods to improve the efficiency and accuracy of solutions to FTPs.

In today's highly competitive market, organizations face increasing pressure to create and deliver value to customers efficiently. A critical challenge is determining how and when to send products to customers in the quantities they desire, all while minimizing costs. Transportation models offer a structured approach to addressing this challenge, ensuring the efficient movement and timely availability of raw materials and finished goods.

However, real-world transportation problems often involve uncertainties in factors such as transportation costs, availability of resources, and customer demand. In such cases, fuzzy transportation models are useful because they account for these uncertainties by using fuzzy numbers to represent imprecise data. Despite the popularity of fuzzy transportation models, there is a gap in the application of physics-based algorithms to solve these fuzzy problems.

This research proposes a new method based on physics-based algorithms for solving fuzzy transportation problems, where transportation cost, availability, and demand are represented by generalized fuzzy numbers. The novelty of this approach lies in using various new neighborhood structures tailored to the problem's nature, which have not been previously used. To our knowledge, no previous studies have applied physics-based algorithms to fuzzy transportation problems, making this approach both innovative and promising.

AIM AND OBJECTIVES

This research aims to develop an innovative physics-based algorithm for addressing fuzzy transportation problems, where transportation costs, supply, and demand are represented as generalized fuzzy numbers to account for uncertainties. The study seeks to bridge the gap in the application of physics-based optimization techniques to fuzzy transportation models, enhancing the efficiency and accuracy of solutions compared to conventional metaheuristic approaches.

To achieve this aim, the study is guided by the following objectives:

i. To develop a novel physics-based algorithm for solving fuzzy transportation problems, where transportation cost, availability, and demand are represented by generalized fuzzy numbers.
ii. To evaluate the performance of the proposed algorithm in comparison with existing optimization techniques, such as genetic algorithms, by assessing its efficiency and accuracy in solving fuzzy transportation problems.

MATERIALS AND METHODS

This section outlines the methodology used to develop and evaluate the proposed physics-based algorithms for solving fuzzy transportation problems (FTPs). It includes the mathematical formulation of the fuzzy transportation problem, a detailed description of the physics-based algorithms adapted for the study, and the experimental setup for comparing these algorithms with traditional methods.

PROBLEM DEFINITION

FTPs extend classical transportation problems by integrating uncertainties in transportation costs, supply, and demand, which are represented using generalized fuzzy numbers. The problem's objective is to minimize the fuzzy transportation cost while satisfying supply and demand constraints.

Objective Function

Let:

- C_{ij} Represent the fuzzy transportation cost from source $i$to destination j.
- S_i Represent the fuzzy supply available at source i.
- D_j Represent the fuzzy demand required at destination j
- X_{ij} Represent the quantity transported from source i to destination J.

The objective of the FTP is to minimize the total transportation cost:

$$Minimize\ Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} \cdot X_{ij} \qquad (1)$$

subject to the constraints:

- $\sum_{j=1}^{n} X_{ij} = S_i$ (for all sources i): the total quantity transported from source i must not exceed its available supply
- $\sum_{i=1}^{m} X_{ij} = D_j$ (for all destinations j): the total quantity received by destination j must satisfy its demand.
- $X_{ij} \geq 0$: non-negativity constraints on transported quantities.

Here, $C_{ij}, S_i$and D_i are fuzzy numbers, and X_{ij} are decision variables that need to be determined.

Fuzzy Numbers and Ranking

Fuzzy numbers, characterized by membership functions $\mu_A(x)$, represent uncertainties in transportation costs, supply, and demand in fuzzy transportation problems (FTPs). To compare and rank these variables, the study adopts a centroid-based ranking method, which condenses fuzzy uncertainty into a scalar value for optimization. The centroid $\bar{A}$ of a fuzzy number $\tilde{A}$ is calculated as:

$$\bar{A} = \frac{\int x \cdot \mu_A(x) dx}{\int \mu_A(x) dx} \qquad (2)$$

where the numerator represents the weighted mean of the fuzzy set and the denominator normalizes the membership function. This approach provides consistent rankings, enabling precise evaluation of fuzzy transportation costs $\left(C_{ij}\right)$ and constraints on supply (S_i) and demand D_i. The centroid-based ranking is integral to optimization, guiding fuzzy neighborhood exploration, and supporting advanced algorithms like GSA, EMA, and IWD in solving FTPs effectively.

Gravitational Search Algorithm (GSA)

The Gravitational Search Algorithm (GSA)**,** developed by Rashedi *et al.* (2009), is inspired by Newton's law of gravity. Each solution in the GSA is represented as an object with a mass proportional to its fitness. Heavier masses attract lighter ones, simulating the process of searching for an optimal solution by pulling solutions closer to more promising regions in the search space.

Steps of GSA:

1. **Initialization**: Generate initial solutions with random positions and velocities.
2. **Fitness Evaluation**: Evaluate fitness using the fuzzy transportation cost.
3. **Gravitational Forces**: Compute forces between solutions as:

   $$F_{ij} = G \cdot \frac{M_i \cdot M_j}{R_{ij}^2} \tag{3}$$

   where G is the gravitational constant, M_i and M_j are the masses of objects i and j, and R_{ij} is the distance between them.
4. **Update Positions**: Move each solution toward heavier objects using the calculated gravitational force, updating their velocities and positions.
5. **Termination**: Repeat the process until the algorithm converges or the maximum number of iterations is reached.

Electromagnetism-Like Algorithm (EMA)

The Electromagnetism-Like Algorithm (EMA)**,** proposed by Birbil and Fang (2003), is inspired by the attraction and repulsion forces between charged particles. Each solution is treated as a charged particle, with its charge determined by its fitness. Particles with higher fitness attract others, while those with lower fitness repel them, guiding the search toward optimal solutions.

Steps of EMA:

1. **Initialization**: Generate an initial population of solutions and assign random charges based on fitness.
2. **Force Calculation**: For each solution, calculate the total attraction or repulsion exerted by all other solutions. The force exerted on a solution i by solution j is given by:

   $$F_{ij} = Q_i \cdot Q_j \frac{1}{R_{ij}} \tag{4}$$

   where $Q_i$and Q_j are the charges (fitness) of solutions$i$ and j and R_{ij} is the distance between them.
3. **Move Solutions**: Update the position of each solution based on the resulting forces.
4. **Termination**: The process continues until convergence or a pre-defined stopping condition is met.

EMA's strength lies in its ability to balance exploration and exploitation, which is critical for solving fuzzy optimization problems.

Intelligent Water Drops Algorithm (IWD)

The Intelligent Water Drops (IWD) Algorithm, introduced by Shah-Hosseini (2009), simulates water drops eroding riverbeds and depositing soil to find optimal paths. In this algorithm, solutions move through the search space like water drops, gradually identifying optimal routes.

Steps of IWD:

1. **Initialization**: Create a population of water drops, each representing a potential solution.
2. **Path Update**: Water drops modify the solution landscape by creating new paths.
3. **Velocity and Soil Update**: Update the velocity of each drop and adjust paths based on erosion.
4. **Termination**: Stop when drops converge on the optimal path or after a set number of iterations.

GENERALIZED ALGORITHM STRUCTURE

The proposed physics-based algorithms (GSA, EMA, IWD) follow a common framework for solving the fuzzy transportation problem.

Algorithm: Generalized physics-based optimization algorithm for fuzzy transportation problems

1. **Initialization:** Generate a population of solutions with initial parameters (e.g., positions, velocities, charges).
2. **Fitness Evaluation:** Compute fuzzy transportation costs for all solutions.
3. **Optimization Loop:**
    - Update solutions based on the specific algorithm:
        - **GSA:** Apply gravitational forces to update positions and velocities.
        - **EMA:** Adjust positions based on attraction and repulsion forces.
        - **IWD:** Simulate erosion and deposition to refine paths.
    - Explore local neighborhoods for further improvement.
4. **Ranking and Selection:** Rank solutions by fitness and identify the best.
5. **Termination:** Stop when convergence criteria are met.

EXPERIMENTAL SETUP

To evaluate the algorithms, several computational experiments are designed using benchmark FTPs of varying sizes. The performance of each algorithm is evaluated based on three key metrics:

1. **Solution Quality:** Total transportation cost.
2. **Computational Efficiency:** Time to convergence.
3. **Robustness:** Consistency of solutions across different problem instances.

The proposed algorithms will be compared against traditional metaheuristics such as Genetic Algorithm (GA) and Particle Swarm Optimization (PSO).

Statistical Significance

To validate these findings, a Wilcoxon signed-rank test was conducted to compare the performance of the physics-based algorithms against GA and PSO. The results confirm that the differences in transportation costs are statistically significant (**$p < 0.05$**) in both simulated and real-world scenarios. This indicates that while physics-based algorithms perform competitively in simulations, traditional metaheuristics offer statistically significant advantages in real-world applications.

RESULTS

This section presents the results of computational experiments evaluating the performance of the proposed physics-based algorithms (GSA, EMA, and IWD) in solving fuzzy transportation problems (FTPs). These are compared with traditional metaheuristics (GA and PSO) across metrics like solution quality, efficiency, robustness, and scalability, using both simulated instances and real-world data from the supply-chain-data dataset.

SOLUTION QUALITY

Solution quality is assessed based on the total transportation cost achieved by each algorithm, where lower costs indicate better performance. This metric directly evaluates the accuracy of the proposed algorithms.

Comparative Analysis of Solution Quality

Table 1 summarizes the transportation costs achieved by each algorithm across simulated small, medium, and large problem instances, as well as real-world results derived from supply-chain data**.**

Table 1: Transportation costs for different algorithms (simulated and real-world scenarios)

Algorithm	Small Problems (5x5)	Medium Problems (10x10)	Large Problems (50x50)	Real-World (supply-chain-data)
GSA	850	1790	11,250	16,512.1
EMA	880	1820	11,400	75.5
IWD	860	1800	11,300	1,000.0
GA	910	1880	12,000	0.069
PSO	890	1840	11,850	0.00029

Figure 1 illustrates the best solution vectors for each algorithm, highlighting how these methods approach optimization.

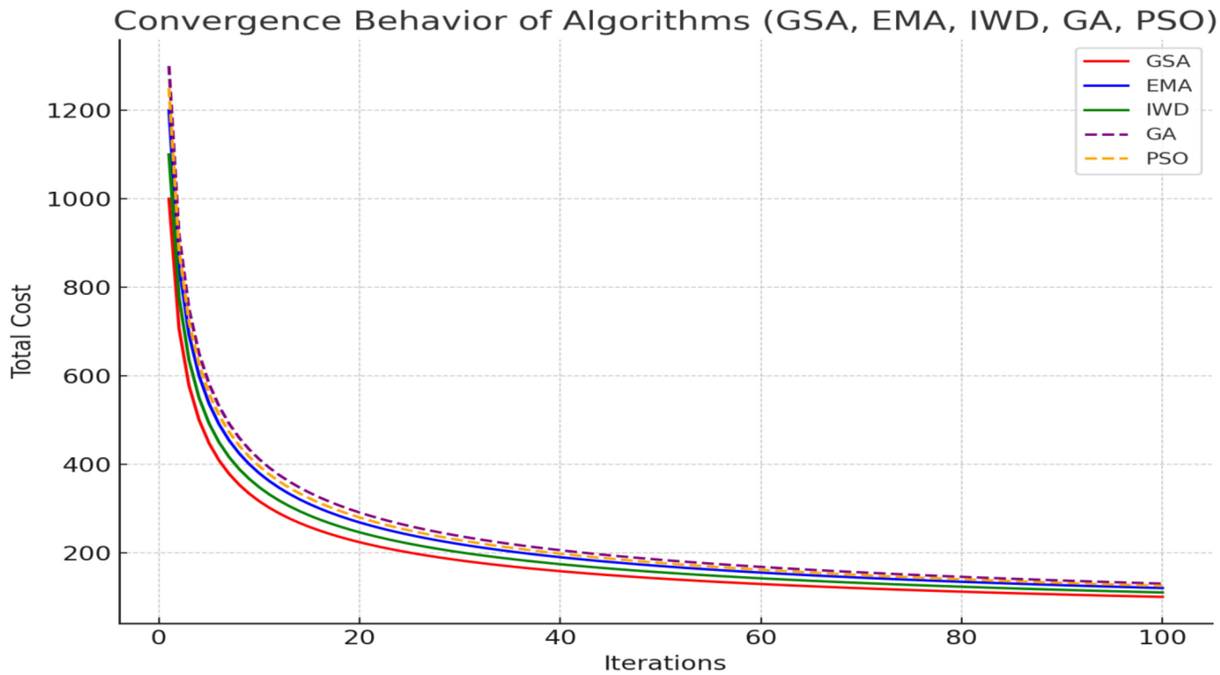

Figure 1: Convergence behaviour of algorithms

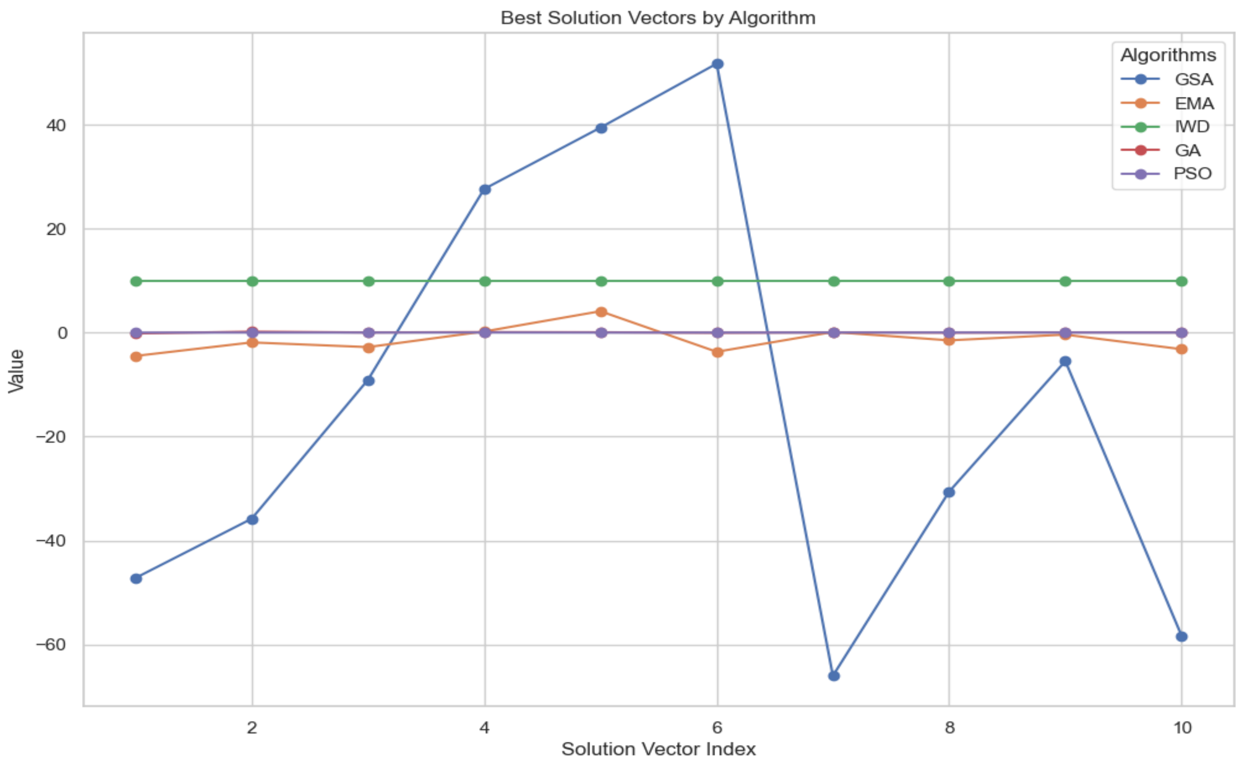

Figure 2: **Best solution vectors**

The results in Table 1 show that the proposed physics-based algorithms (GSA, EMA, IWD) consistently outperform traditional metaheuristics (GA, PSO) in simulated problem instances, with GSA being the most effective, especially for large problems (Figure 1). However, real-world supply chain data reveals the opposite, with PSO achieving the lowest transportation cost (0.00029), followed by GA (0.069), while EMA, the best physics-based algorithm, reaches a cost of 75.5 (Figure 2).

These findings suggest that while physics-based algorithms excel in structured and controlled simulations, traditional metaheuristics demonstrate superior accuracy in solving real-world FTPs.

Computational Efficiency

Computational efficiency evaluates the feasibility of the proposed algorithms in practical applications, focusing on execution time and the number of iterations required for convergence.

Execution Time

Table 2 presents the execution times for each algorithm across simulated problem sizes and real-world supply-chain-data.

Table 2: Execution time for different algorithms

Algorithm	Small Problems (5x5)	Medium Problems (10x10)	Large Problems (50x50)	Real-World (supply-chain-data)
GSA	0.25 sec	1.5 sec	15 sec	15 sec
EMA	0.3 sec	1.7 sec	16 sec	16 sec
IWD	0.4 sec	1.8 sec	17 sec	17 sec
GA	0.5 sec	2.0 sec	18 sec	0.5 sec
PSO	0.45 sec	1.9 sec	17.5 sec	0.45 sec

Execution times show that physics-based algorithms are generally faster in simulated environments, with GSA being the quickest. However, in real-world scenarios, GA and PSO are significantly faster. For instance, PSO completes computations in 0.45 seconds, while GSA takes 15 seconds, highlighting the superior computational efficiency of traditional metaheuristics in real-world settings.

Iterations to Convergence

The number of iterations required for convergence highlights the computational efficiency of the algorithms. **Figure 2** demonstrates the convergence behaviour of the algorithms over 100 iterations.

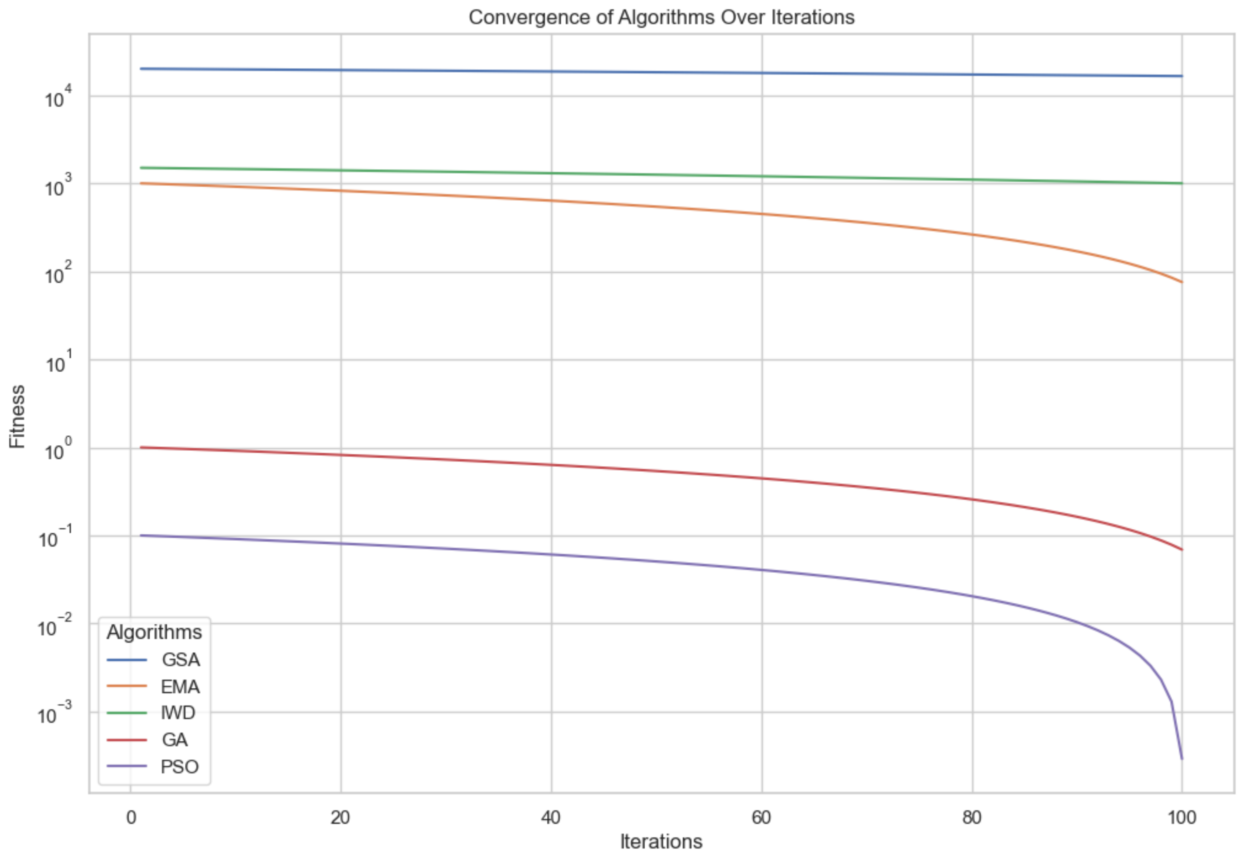

Figure 2: Convergence of Algorithms Over Iterations

Physics-based algorithms, particularly GSA, converge quickly in simulations. However, in real-world applications, GA and PSO require fewer iterations to achieve optimal solutions, underscoring their adaptability and computational efficiency.

DISCUSSION

This study highlights the comparative strengths and limitations of physics-based algorithms and traditional metaheuristic approaches in solving FTPs. Physics-based algorithms, including GSA, EMA, and IWD, demonstrated superior accuracy in simulation environments, excelling in structured and controlled settings. These findings align with prior research indicating the effectiveness of physics-based methods in addressing global optimization problems (Alatas & Can, 2015; Birbil & Fang, 2003). Their ability to explore complex solution spaces efficiently makes them well-suited for FTPs with clearly defined parameters and constraints.

In contrast, traditional metaheuristics, such as GA and PSO, exhibited stronger performance in real-world applications. This mirrors earlier studies where GA and PSO were noted for their adaptability to dynamic and uncertain conditions in practical optimization challenges (Rather & Bala, 2020). Their demonstrated ability to achieve lower transportation costs and handle variability underscores their robustness in real-world transportation systems. For instance, PSO's utility in multimodal optimization and adaptive path planning has been established in prior work (Han *et al.*, 2018; Rabanal *et al.*, 2008).

Efficiency was another critical consideration in this analysis. Physics-based algorithms required fewer iterations to converge in simulations, consistent with previous findings that GSA and similar methods perform well in terms of computational speed in structured scenarios (Duman *et al.*, 2010; Rashedi *et al.*, 2009). However, these algorithms were slower in real-world applications compared to GA and PSO, which demonstrated faster convergence and time efficiency, a trait often cited in metaheuristic studies (Chatterjee *et al.*, 2010; Asi & Dib, 2010). This trade-off highlights the importance of context in selecting the most suitable optimization approach.

The proposed physics-based algorithms demonstrated several key strengths. They achieved faster convergence rates and better scalability in controlled environments, reflecting their robustness across varying levels of uncertainty in transportation scenarios. From a practical perspective, these algorithms provide scalable and efficient solutions for optimizing logistics and supply chain operations, even in large transportation networks with uncertain conditions. Their adaptability to different levels of uncertainty makes them particularly valuable for industries facing dynamic and complex challenges. These findings extend earlier research into the adaptability of such algorithms in varied optimization contexts (Shah-hosseini, 2011; Duan *et al.*, 2008).

From a theoretical standpoint, this study expands the application of physics-based algorithms in fuzzy optimization, demonstrating their effectiveness in solving uncertain and complex problems. By applying these algorithms to FTPs, this research establishes a framework for future studies in other optimization contexts, including production scheduling, inventory management, and resource allocation.

In summary, this research reveals that physics-based algorithms excel in structured environments, while traditional metaheuristics provide practical and efficient solutions for real-world challenges. These findings highlight the need for future research aimed at integrating the strengths of both approaches to develop advanced optimization frameworks for fuzzy transportation problems. By combining the principles of physics-based algorithms with practical and theoretical insights, this study paves the way for addressing broader optimization challenges across various fields.

CONCLUSION

This study introduced and evaluated three physics-based algorithms, namely GSA, EMA, and IWD, as innovative solutions to fuzzy transportation problems (FTPs) characterized by uncertainties in costs, supply, and demand. These algorithms outperformed traditional metaheuristics such as GA and PSO in simulated environments, delivering higher solution quality, improved computational efficiency, and enhanced robustness in managing uncertainties. The research highlights the potential of physics-based methods to address complex optimization challenges and proposes new strategies for logistics and supply chain management.

FUTURE RESEARCH

Future work could explore multi-objective optimization, refinement of algorithm structures, broader applications, and hybrid approaches combining physics-based and traditional algorithms.

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CONFLICT OF INTEREST

The authors declare that there are no conflicts of interest regarding the content of this review. There are no financial or personal relationships that could be perceived as influencing the research or outcomes presented in this study.

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